12-6 Permutations and Combinations

Vocabulary

Review

1. *Combine* the two sentences below to form one sentence.

   I will order a hamburger. I will pay for it with a $1 bill.

   ____

2. *Combine* the bowling scores below to find the total for the three games.

   103 149 97 Total: ____

***Combine* like terms in each expression.**

3. $3x + 5x =$ ____
4. $4a^2 + 3a - 5 - 6a^2 - 2a =$ ____
5. $8x + 2y - x =$ ____
6. $2c^2 + 5cd - d^2 + 3d =$ ____

Vocabulary Builder

permute (verb) pur MYOOT

Related Words: order (verb, noun), permutation (noun)

Definition: To **permute** a list of numbers means to rearrange the order or sequence of the numbers.

Math Usage: Sometimes you want to know how many ways you can **permute** a set of objects. Each ordered arrangement of the objects is called a *permutation*.

Use Your Vocabulary

7. Underline the correct word to complete each sentence.

   You can permute / permutation the digits in the number 394 to create different numbers.

   The number 934 is a permute / permutation of the digits 3, 9, and 4.

8. *Permute* these race results to show another possible ordered arrangement.

   first place: Rita second place: David third place: Beth

   first place: ____ second place: ____ third place: ____

take note — Key Concept Multiplication Counting Principle

If there are m ways to make a first selection and n ways to make a second selection, then there are $m \cdot n$ ways to make the two selections.

9. There are 4 types of bread and 5 types of sandwich meat. You choose one type of bread and one type of sandwich meat for your sandwich. How many different sandwiches are possible?

$4 \cdot$ ____ $=$ ____, so ____ different sandwiches are possible.

10. You have 9 shirts and 4 pairs of pants. How many different outfits can you make?

____ $\cdot$ ____ $=$ ____, so you can make ____ different outfits.

A **permutation** is an arrangement of objects in a specific order. Here are the possible permutations of the letters A, B, and C without repeating any letters.

ABC ACB BAC BCA CAB CBA

Problem 2 Finding Permutations

Got It? A swimming pool has 8 lanes. In how many ways can 8 swimmers be assigned lanes for a race?

11. Working from left to right, write how many choices there are for a swimmer to be assigned to each lane. (*Hint:* When you determine the number of choices for a lane, assume that a swimmer has been chosen for each lane to the left.)

Lane 1	Lane 2	Lane 3	Lane 4	Lane 5	Lane 6	Lane 7	Lane 8

12. Write the missing factors below to show how many ways 8 swimmers can be assigned lanes for a race.

$8 \cdot$ ____ $\cdot$ ____ $\cdot$ ____ $\cdot$ ____ $\cdot$ ____ $\cdot$ ____ $\cdot$ ____

13. There are ____ ways 8 swimmers can be assigned lanes.

A shorter way to write the product in Problem 2 is 8!, read "eight factorial." For any positive integer n, the expression ***n* factorial** is written as $n!$ and is the product of the integers from n down to 1. The value of 0! is defined to be 1.

You can use factorials to write a formula for the number of permutations of n objects arranged r at a time.

take note — Key Concept Permutation Notation

The expression ${}_nP_r$ represents the number of permutations of n objects arranged r at a time.

$${}_nP_r = \frac{n!}{(n-r)!}$$

Example ${}_8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 56$

14. Find the number of permutations of 5 objects arranged 3 at a time.

$$ {}_5P_3 = \frac{\square\,!}{(\square - \square)!} = \frac{\square\,!}{\square\,!} = \frac{\square \cdot \square \cdot \square \cdot \square \cdot \square}{\square \cdot \square} = \square $$

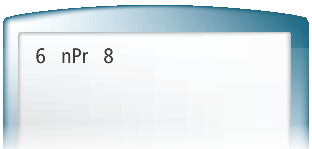
Problem 3 Using Permutation Notation

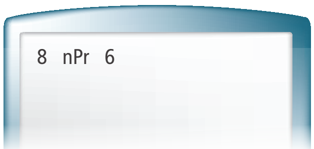
Got It? **There are 6 students in a classroom with 8 desks. How many possible seating arrangements are there?**

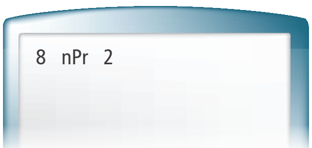
15. Circle the expression that will help you solve the problem.

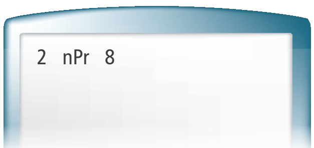
${}_6P_8$ ${}_8P_6$ ${}_8P_2$ ${}_2P_8$

16. Circle the graphing calculator screen that shows the problem.









17. The number of possible seating arrangements is $\square$.

A **combination** is a selection of objects without regard to order. For example, if you are selecting two side dishes from a list of five, the order in which you choose the side dishes does not matter.

take note

Key Concept Combination Notation

The expression ${}_nC_r$ represents the number of combinations of n objects chosen r at a time.

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

Example $ {}_8C_2 = \frac{8!}{2!(8-2)!} = \frac{8!}{2!6!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 28 $

18. Find the number of combinations of 4 objects chosen 3 at a time.

$$ {}_4C_3 = \frac{\square\,!}{\square\,!(\square - \square)!} = \frac{\square\,!}{\square\,!\;\square\,!} = \frac{\square \cdot \square \cdot \square \cdot \square}{(\square \cdot \square \cdot \square)(\square)} = \square $$

Problem 4 Using Combination Notation

Got It? **In how many different ways can you choose 3 types of flowers for a bouquet from a selection of 15 types of flowers?**

19. Does it matter in which order you choose the three types of flowers? Yes / No

20. Circle the expression that will help you solve the problem.

${}_{15}P_3$ ${}_{15}C_3$ ${}_{3}C_{15}$ ${}_{3}P_{15}$

21. Find the number of possible ways to choose the three types of flowers.

22. There are ____ possible ways to choose the three types of flowers.

Lesson Check • Do you UNDERSTAND?

Vocabulary **Would you use *permutations* or *combinations* to find the number of possible arrangements of 10 students in a line? Why?**

23. Underline the correct word(s) to complete the sentence.

If the first two students in line switch positions, the order of the 10 students is / is not changed.

24. Would you use permutations or combinations to find the number of possible arrangements of 10 students in a line? Explain.

____

____

Math Success

Check off the vocabulary words that you understand.

☐ Multiplication Counting Principle ☐ permutation ☐ *n* factorial ☐ combination

Rate how well you can *find permutations and combinations.*

Need to review 0 2 4 6 8 10 Now I get it!